ERRATA FOR "CANONICAL SUBGROUPS VIA BREUIL-KISIN MODULES"

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The proof of [1, Proposition 4.3 (2)] is incorrect. In page 950 line 1–2, the author claims that the assertion (2) of the proposition is deduced from "The uniqueness of the canonical subgroup in Theorem 3.1 (1)". However, the truncated Barsotti-Tate group $p^{-1}\mathcal{D}/\mathcal{D}$ has Hodge weight $p^{-1}w$, while the subgroup scheme $\mathcal{G}[p]/\mathcal{D}$ coincides with the Frobenius kernel only on modulo $m_K^{\geqslant 1-w}$ and we cannot apply the uniqueness assertion of Theorem 3.1 (1). Here we give a correct proof of Proposition 4.3 (2).

Lemma 0.1. *Let $K/\mathbb{Q}_p$ be a finite extension. Let $\mathcal{G}$ be a truncated Barsotti-Tate group over $\mathcal{O}_K$ of level one, height h, dimension d with $0 < d < h$ and Hodge height w. Let $\mathcal{H}$ be a finite flat closed subgroup scheme of $\mathcal{G}$ over $\mathcal{O}_K$ of height d. Put $f = \deg(\mathcal{G}/\mathcal{H})$. Assume $f < 1/2$. Then $\mathrm{Hdg}(\mathcal{G}) = f$ and $\mathcal{H}$ is the canonical subgroup of $\mathcal{G}$.*

Proof. Let e be the absolute ramification index of K. Let $\mathfrak{L}$, $\mathfrak{M}$ and $\mathfrak{N}$ be the Breuil-Kisin modules associated to $\mathcal{G}/\mathcal{H}$, $\mathcal{G}$ and $\mathcal{H}$, respectively. We denote the dual of $\mathfrak{M}$ by $\mathfrak{M}^\vee$. The $k[[u]]$-module $\mathfrak{M}_1^\vee = \mathfrak{M}^\vee/u^e\mathfrak{M}^\vee$ has a natural structure of a φ-module induced from that of $\mathfrak{M}^\vee$ and put

$$\mathrm{Fil}^1\mathfrak{M}_1^\vee = \mathrm{Im}(1 \otimes \varphi_{\mathfrak{M}_1^\vee} : \varphi^*\mathfrak{M}_1^\vee \to \mathfrak{M}_1^\vee).$$

Since $\mathcal{G}^\vee$ is of dimension $h-d$, this is a free $k[[u]]/(u^e)$-module of rank d, and also a direct summand of the $k[[u]]$-module $\mathfrak{M}_1^\vee$ [1, §2.3]. We write v_u for the u-adic valuation on $k[[u]]$ normalized as $v_u(u) = 1$. By [1, Lemma 2.5 (3)], we have $f = e^{-1}v_u(\det(\varphi_{\mathfrak{L}}))$. By the definition of the dual $\mathfrak{L}^\vee$, the image $\varphi_{\mathfrak{L}^\vee}(\mathfrak{L}^\vee)$ is contained in $u^{e(1-f)}\mathfrak{L}^\vee$.

We have an exact sequence of φ-modules over $k[[u]]/(u^{e(1-f)})$

$$0 \longrightarrow \mathfrak{N}^\vee/u^{e(1-f)}\mathfrak{N}^\vee \longrightarrow \mathfrak{M}^\vee/u^{e(1-f)}\mathfrak{M}^\vee \longrightarrow \mathfrak{L}^\vee/u^{e(1-f)}\mathfrak{L}^\vee \longrightarrow 0$$

and the Frobenius map of the φ-module $\mathfrak{L}^\vee/u^{e(1-f)}\mathfrak{L}^\vee$ is zero. Thus the Frobenius map of $\mathfrak{M}^\vee/u^{e(1-f)}\mathfrak{M}^\vee$ factors through $\mathfrak{N}^\vee/u^{e(1-f)}\mathfrak{N}^\vee$, and we obtain the inclusion

$$\mathrm{Fil}^1\mathfrak{M}_1^\vee/u^{e(1-f)}\mathrm{Fil}^1\mathfrak{M}_1^\vee \subseteq \mathfrak{N}_1^\vee/u^{e(1-f)}\mathfrak{N}_1^\vee.$$

Since the both sides are of length $de(1-f)$ as $k[[u]]$-modules, the equality

$$\mathrm{Fil}^1\mathfrak{M}_1^\vee/u^{e(1-f)}\mathrm{Fil}^1\mathfrak{M}_1^\vee = \mathfrak{N}_1^\vee/u^{e(1-f)}\mathfrak{N}_1^\vee$$

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holds, from which we obtain

$$\min\{1-f, e^{-1}v_u(\det(\varphi_{\mathrm{Fil}^1\mathfrak{M}_1^\vee}))\} = \min\{1-f, e^{-1}v_u(\det(\varphi_{\mathfrak{N}^\vee}))\}.$$

On the other hand, we have

$$v_u(\det(\varphi_{\mathfrak{N}^\vee})) = e\deg(\mathcal{H}^\vee) = e(d-\deg(\mathcal{H})) = e(d-(d-f)) = ef.$$

By the assumption $f < 1/2$, we have $f < 1-f$ and the above equality implies

$$w = \mathrm{Hdg}(\mathcal{G}) = \mathrm{Hdg}(\mathcal{G}^\vee) = e^{-1}v_u(\det(\varphi_{\mathrm{Fil}^1\mathfrak{M}_1^\vee})) = f < 1/2.$$

By [1, Theorem 3.1], the truncated Barsotti-Tate groups $\mathcal{G}$ and $\mathcal{G}^\vee$ have the canonical subgroups of level one. Note that the canonical subgroup of $\mathcal{G}^\vee$ is constructed in [1, Theorem 3.1] as the finite flat closed subgroup scheme $\mathcal{C}'$ of $\mathcal{G}^\vee$ such that the quotient $\mathcal{G}^\vee/\mathcal{C}'$ is associated to the unique lift of $\mathrm{Fil}^1\mathfrak{M}_1^\vee/u^{e(1-w)}\mathrm{Fil}^1\mathfrak{M}_1^\vee$ in $\mathfrak{M}^\vee$ as a Breuil-Kisin submodule. This implies that $(\mathcal{G}/\mathcal{H})^\vee$ is the canonical subgroup of $\mathcal{G}^\vee$. By [1, Theorem 3.1 (b)], we see that $\mathcal{H}$ is the canonical subgroup of $\mathcal{G}$. $\square$

Now we prove [1, Proposition 4.3 (2)]. Put $w = \mathrm{Hdg}(\mathcal{G})$. We are assuming $w < 1/2$. Since we have a generic isomorphism $\mathcal{C} \to \mathcal{G}[p]/\mathcal{D}$, [1, Theorem 3.1 (a)] implies

$$d - w = \deg(\mathcal{C}) \leq \deg(\mathcal{G}[p]/\mathcal{D}) = d - \deg(\mathcal{D})$$

and thus

$$\deg((p^{-1}\mathcal{D}/\mathcal{D})/(\mathcal{G}[p]/\mathcal{D})) = \deg(\mathcal{D}) \leq w < 1/2.$$

Therefore we can apply Lemma 0.1 and we see that $\mathcal{G}[p]/\mathcal{D}$ is the canonical subgroup of $p^{-1}\mathcal{D}/\mathcal{D}$.

References

[1] S. Hattoti: *Canonical subgroups via Breuil-Kisin modules*, Math. Z. **274** (2013), 933–953.

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